Restoration of beta-decay events that have occurred during the detection system's dead time

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Dead time of the apparatus used for particle detection affects every measurement of nuclear half-life, while its nature and extent may not be exactly known. Consequently, the data obtained in such measurements yield results that may depend on some educated estimates of the dead-time effects. These effects increase when the product of the dead time per event and the event rate expected in the absence of dead time (*i.e.* the ideal rate) increases, in which case the errors in the estimates have a greater effect on the accuracy of the measured half-lives. Therefore, there is a need for a proven standard method of measurement and analysis that would provide the best accuracy and precision of the results for a given number of measured events.

Such a method has been developed recently [1]. It requires a measurement of the time of each particle-detection event, which can easily be accomplished using a suitable time-to-digital converter (TDC) [1]. The key element in the analysis is the imposition of a known, sufficiently large extending dead time to the measured sequence of events in order to produce a set of surviving events for which the effects of the actual dead time are completely obliterated by the effects of the imposed dead time. As a result, the dead time following each surviving event and the live time preceding each surviving event are known exactly, which provides for an exact statistical analysis of the time intervals between consecutive surviving events. We validated this method by using simulated data for the beta-decay of ^{26m}Al (at half-life $T_{1/2}$ = 6.3452 s [2]), with an assumed constant background event-rate of B = 1 s^{-1} and an initial decay rate, A, ranging from 100 s^{-1} to 100,000 s^{-1}. The imposed extending dead-time per event, τ_e, ranged from 2 μs to 512 μs, so that $A\tau_e$ ranged from 2×10^{-4} to as high as 51.2.

However, it turns out that the time it takes to analyze the data using this method for a given total number of events increases drastically as the event rate decreases. This is mainly because in this case the number of events per sample decreases, so that an increasingly larger number of samples must be analyzed. While the latter leads only to the need for increased computing power and memory size, the former presents an additional problem: As the average number of events per sample decreases and becomes too small, the distribution of the maximum-likelihood values of the fitting-function parameters obtained in the analysis of each sample broadens to the point at which it becomes affected by the physical restrictions of the problem, such as the requirement that all parameters have positive values. This leads to an increased systematic error and a biased result. Unfortunately, systematically ignoring the affected samples has the same kind of an effect.

A traditional solution to this problem would be to (a) produce a decay spectrum (*i.e.*, a time histogram) of the surviving events for each sample, (b) correct the number of events in each channel of each decay spectrum for dead-time effects, and (c) combine channel-wise the resulting dead-time-corrected (dead-time-free) decay spectra into a single decay spectrum for an analysis to determine the nuclear half-life. While parts (a) and (c) of this procedure are straightforward, part (b) may only seem straightforward because the live- and dead-times associated with each surviving event are assumed to be known exactly. To the best of our knowledge, none of the existing published methods that could be applied here was proven to be unbiased, numerically problem-free, and applicable to the cases involving low event rates and large dead-time corrections. Thus, this problem is far from being trivial.

For the method in which the dead-time-corrected number of events in a given channel is obtained by dividing the observed number of events in that channel with the total channel live time and multiplying the result with the channel duration, the extent of the problem is illustrated in Fig. 1a, which is based a data set with approximately one million events simulated at a constant ideal event rate $\rho=10s^{-1}$, and partitioned into one thousand samples, each having 100 channels spanning the duration of 100 s. The imposed extendable dead time per event τ_e was varied from 0 to 120 ms. Evidently, the dead-time-corrected number of events divided by the actual number of events in the absence of dead time is increasingly greater than the expected value of 1 as the dead time increases, reaching the value of as high as 1.22 at τ_e = 100 ms. A similar behavior is expected in the case of constant τ_e and changing ρ, since for the given data set the effect depends on the observed event rate, which in turn depends on $\rho\tau_e$. It should be noted, though, that the same method of dead-time correction yields the expected (and desired) result, as

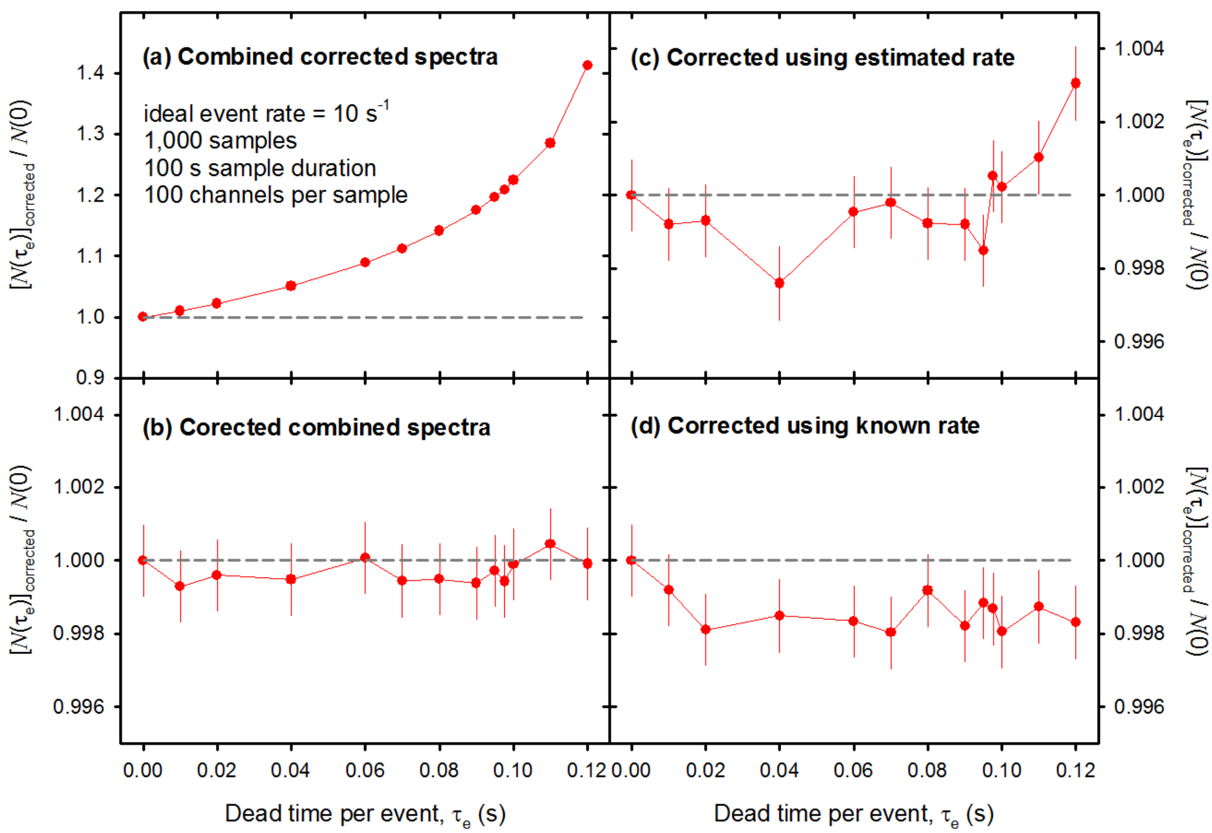

FIG. 1. Total dead-time-corrected number of events $[N(\tau_e)]_{corrected}$ divided by the total number of events in the absence of dead time $N(0)$ for a simulated data set described in the legend. (a) Live-time-fraction correction was applied to one channel of one sample at a time. (b) Live-time-fraction correction was applied to one channel at a time after all samples had been combined. (c) Simulated dead-time correction was applied to one channel of one sample at a time, using the ideal event rate estimated from the sum of three consecutive live-time intervals. (d) Simulated dead-time correction was applied to one channel of one sample at a time based on the ideal event rate actually used in the simulations.

shown in Fig. 1b, if part (c) of the procedure is applied before part (b). This shows that, for a given total number of events, the bias of this method increases as the observed (effective) event rate decreases. Unfortunately, the decay spectra obtained from each sample can be combined first and the combination dead-time corrected next only if the ideal event rate function is the same for all samples, which cannot be guaranteed in the real measurements of nuclear decay.

Therefore, a new approach to the problem is proposed here, in which (*i*) the events lost due to dead time are replaced by a statistically equivalent set of events and inserted appropriately into the sequence of surviving events, thus producing a statistically correct dead-time-free sequence of events. This would be followed by (*ii*) producing a decay spectrum of these events for each sample and (*iii*) combining channel-wise these decay spectra into a single decay spectrum for analysis. While parts (*ii*) and (*iii*) of this new procedure are straightforward, part (*i*) has yet to be developed. The key requirement is that the dead-time correction must be based solely on the known live-time and dead-time intervals associated with the surviving events, and specifically, without any advanced knowledge of the ideal rate or its time-dependence.

Given that the number of events lost due to the imposed extending dead time per event τ_e depends directly only on the duration of the dead-time interval following each surviving event (which is known exactly if τ_e is sufficiently large) and the ideal rate at the time of each surviving event (which is not known *a priori*), the latter quantity must be estimated based on the known live times preceding each surviving event. This is possible because, at a given ideal event rate ρ, the distribution of these live times is identical to the distribution dp_1/dt of the times between consecutive events in the absence of dead time, and is given by

$$dp_1/dx_1 = \exp(-x_1) \,, \tag{1}$$

where

$$x_1 = \rho t \,. \tag{2}$$

However, even though the mean value of x_1 equals 1, $1/t$ is not a good statistical estimate of ρ because the most probable value of dp_1/dx_1 is zero, which means that it is very likely to encounter a live time interval so small that its reciprocal value exceeds that of the actual ideal rate by many orders of magnitude. On the other hand, if n consecutive live times are combined, the distribution of their sum t_n is given by

$$dp_n/dx_n = (x_n)^{n-1} \exp(-x_n) / n! \,, \tag{3}$$

where $x_n = \rho t_n$. This distribution peaks at the value of $(x_n)_{\text{peak}} = n - 1$ and (for $n > 1$) it tends to zero both for extremely small and extremely large values of x_n, thus making $(n - 1)/ t_n$ a good statistical estimate of ρ.

The distribution of p_n gets narrower as n increases, in which case the estimates of ρ improve. However, a large value of n is not practical when ρ is expected to change significantly between the beginning and the end of the time interval x_n. On the other hand, when this is not the case, the live-time intervals to be combined can be sampled using patterns that do not necessarily involve consecutive values.

To illustrate the new method of dead-time correction proposed here, we set n = 3 and use consecutive live-time values. Specifically, we populate the dead-time interval $(t_D)_i$ that follows the surviving event i (for all available values of i) with simulated events, assuming the ideal event rate of 1 / $[(t_L)_i + (t_L)_{i+1} + (t_L)_{i+2}]$, where $(t_L)_i$, $(t_L)_{i+1}$, and $(t_L)_{i+2}$ are, respectively, the live-time intervals preceding the surviving events i, i+1, and i+2. The result is shown in Figure 1c. For comparison, we repeat the same procedure using the ideal event rate used in the simulation and show the result in Figure 1d. Apparently, the bias of the new method of dead-time correction, if any, is commensurate with the expected statistical fluctuations, at least for $\rho\tau_e < 1.1$, and the method works much better than the traditional method based on the channel-by-channel correction, one sample at a time.

[1] V. Horvat and J.C. Hardy, Nucl. Instrum. Methods Phys. Res. **A713**, 19 (2013).
[2] J.C. Hardy and I.S. Towner, Phys. Rev. C **79**, 055502 (2009).